An Energy-Efficient Algorithm For Conflict-Free AGV Routing On A Linear Path Layout

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Abstract: Automated Guided Vehicle (AGV) systems offer flexibility and automation required by Flexible Manufacturing Systems (FMS). Previous research on AGV systems mainly focuses on the performance of the task completion time and the AGV utilization in dispatching and routing. However, as energy shortage and pollution to environment become major concerns and the competition become more intense, the issue of minimizing the energy consumption will unavoidably become an important consideration for the AGV systems. The primary objective of this paper is to provide an energy model for evaluating the energy requirement of the routing algorithms. We present a new linear layout and a routing algorithm. The path layout and the algorithms are analyzed quantitatively for assuring conflict-freedom and the energy efficiency.

1. Introduction

Current AGV systems mainly focus on issues related to minimizing the task completion time and/or maximizing AGV utilization in dispatching and routing [1-7]. Here we address the issue of efficient use of another key resource, namely the energy, aiming to minimize energy consumption of the vehicles during routing. As the energy resource becomes scarce and competition becomes more intense, there is no doubt that minimization of this resource, while maintaining the same level of performance, will be an important consideration for the long term interest of all concerned.

In [6-7], routing on a simple linear layout was considered. The layout consists of two parallel lanes connected by “bridges”. All the Pick up-Drop off stations (or P/D stations for short) and the car park are distributed on the side of one lane. The algorithms proposed in [6-7] can route a batch of vehicles concurrently to carry out P/D jobs without conflicts during the operations of AGVs. We present a new linear layout in which the mirror virtual stations of [6] are now actual stations and the number of bridges is doubled. This modification resulted in slightly better utilization of land space and energy resources and more flexible traffic flow.

Based on this new layout, a routing algorithm is presented in this paper which allows AGVs to carry out P/D jobs concurrently while assuring freedom of conflicts. The critical conditions are also given based on mathematical calculations.

Meanwhile, to account for the energy consumption, we formulate a basic energy model for AGV routing, and the energy efficiency of the algorithm is also analyzed based on this energy model. The analyses show that our algorithm is efficient in both task time as well as energy consumption.

The 1-D, multiple-loop pathway layout turns out to be a popular path configuration in real-world applications [2][4-5]. Our routing algorithm can be a basis for more complicated layouts, such as a mesh-like path topology that often arises in real world applications, e.g. container ports.

The remainder of the paper is organized as follows. The next section describes the design of the linear path layout. The routing algorithm is given in Section 3. In Section 4, we demonstrate that the routing is conflict-free for all vehicles. We also quantify the relationships among certain key parameters of the path layout. In Section 5, we present a basic energy model for AGV routing. In Section 6, the energy efficiency of the algorithm is analyzed in terms of the lower bound of energy requirement for AGVs carrying out the jobs in the linear path layout, and the upper bound of energy required for our routing algorithm. Finally, Section 7 discusses possibilities of relaxing certain constraints before offering concluding remarks.

2. Path design

Fig. 1 shows the path layout, whose details are described as follows:

(a) There are two parallel lanes, namely Lanes L_1 and L_2. There are in total 2n of P/D stations, denoted by Station 1, 2, …, n, distributed on both Lane L_1 and Lane L_2. At each station, there is a buffer that allows one vehicle to stop in order to either pick up or drop off its load. A vehicle can pass by a station along the lane while loading or unloading process of another vehicle is being carried out in the buffer.

(b) There is a vehicle park identified as Station 0 where all AGVs are stationed initially and to which they will return upon completion of all tasks.

(c) There are two “bridges” linking both lanes at each station except for Station 0. A bridge is normally a short lane. And all bridges have the equal length of b.

(d) The distance between Station 0 and Station 1 is p. The distance between any other two adjacent stations is d.

(e) Both the bridges and the traveling lanes are bi-directional, but they are not wide enough to allow more than one vehicle to run side by side at a time.

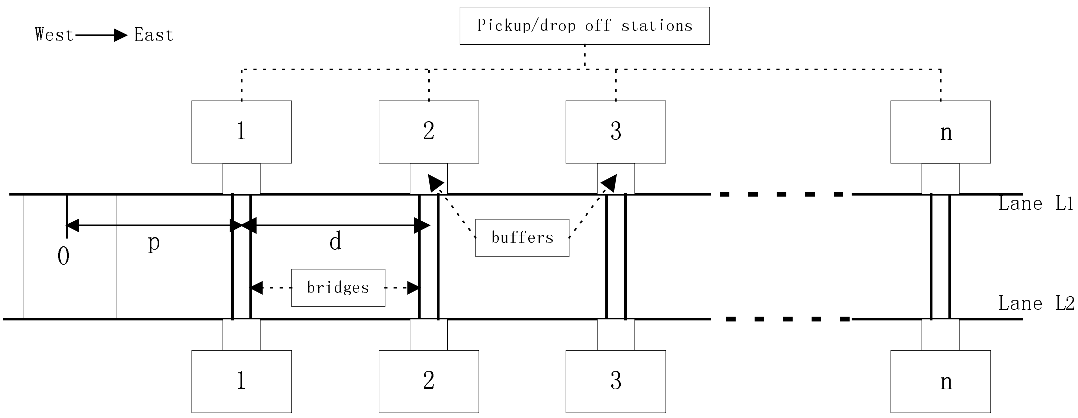

Figure 1. Path Layout

3. Routing algorithm

Given the path layout, we formally define a job set as follows.

Definition 1(Job): *A job is identified by an ordered pair* $((S_i,P_i), (E_i,D_i))$, *where* S_i *and* E_i *represent the lanes of pickup station and drop-off station,* P_i *and* D_i *represent the pickup station and drop-off station position respectively for for* $i=1, 2, \ldots, n$. *Define the value of* $S_i=1$ *(respectively* $E_i = 1$*), if the pickup (resp. drop-off) station is located at Lane* L_1. *Otherwise, define the value of* $S_i=2$ *(resp.* $E_i = 2$*), if the station is at Lane* L_2.

Definition 2(Job Set): *A job set M denoting a set of k jobs, where* $2\leq k\leq n$, *is defined as follows:*

$$M=\{((S_i, P_i), (E_i, D_i)) \mid 1\leq S_i, E_i \leq 2, 1\leq P_i, D_i\leq n, \text{ for } i=1, 2, \ldots, k \}.$$

According to the positions of the origins and destinations of jobs, any given job set M can be divided into four disjoint subsets, denoted by J_1^+, J_1^-, J_2^+, J_2^- respectively, such that

$$J_1^+=\{ ((S_i, P_i), (E_i, D_i)) \mid S_i=1, P_i < D_i) \text{ for } i=1, 2, \ldots, k\}.$$
$$J_1^-=\{ ((S_i, P_i), (E_i, D_i)) \mid S_i=1, P_i > D_i) \text{ for } i=1, 2, \ldots, k\}.$$
$$J_2^+=\{ ((S_i, P_i), (E_i, D_i)) \mid S_i=2, P_i < D_i) \text{ for } i=1, 2, \ldots, k\}.$$
$$J_2^-=\{ ((S_i, P_i), (E_i, D_i)) \mid S_i=2, P_i > D_i) \text{ for } i=1, 2, \ldots, k\}.$$

Accordingly, we have the following notations:

A_1^+ : the set of AGVs that carry out jobs in J_1^+;
A_1^- : the set of AGVs that carry out jobs in J_1^-;
A_2^+ : the set of AGVs that carry out jobs in J_2^+;
A_2^- : the set of AGVs that carry out jobs in J_2^-;
$|J_1^+|$: the number of jobs in J_1^+;
$|J_1^-|$: the number of jobs in J_1^-;
$|J_2^+|$: the number of jobs in J_2^+;
$|J_2^-|$: the number of jobs in J_2^-;

Our assumptions are as follows:

(a) All AGVs travel at the same speed on either Lane L_1 or L_2, but they have to slow down when crossing bridges or making 90-degree turns.
(b) Each job has a distinct origin and also a distinct (but different) destination.
(c) An AGV is given only one job and any job is assigned to only one AGV.

Based on the assumption, the routing algorithm is presented as follows.

Step 1. Let all k AGVs set out one by one at an interval of time from the park along Lane L_1 and L_2. On both lanes, assign the first AGV to go to the furthest pickup station, the second one to the second furthest pickup station, and so forth.

Step 2. Once all AGVs reach their assigned pickup stations, begin the loading processes. When all AGVs are loaded, let them set out simultaneously to their drop-off stations.

Step 3

Case a $|J_2^+|+|J_1^-|\geq|J_2^-|+|J_1^+|$ In this case, let all AGVs in A_2^+ advance along Lane L_2 from west to east, and let all AGVs in A_1^- advance along Lane L_1 from east to west, and let all AGVs in A_2^- first cross the bridges before their pickup stations to reach their pickup mirror stations and then advance along Lane L_1 from east to west, and let all AGVs in A_1^+ first cross bridge to reach their pickup mirror stations and then advance along Lane L_2 from west to east. Go to Step 4.

Case b $|J_2^+|+|J_1^-|<|J_2^-|+|J_1^+|$ In this case, let all AGVs in A_2^- advance along Lane L_2 from east to west, and let all AGVs in A_1^+ advance along Lane L_1 from west to east, and let all AGVs in A_2^+ first cross the bridges before their pickup stations to reach their pickup mirror stations and then advance along Lane L_1 from west to east, and let all AGVs in A_1^- first cross bridge to reach their pickup mirror stations and then advance along Lane L_2 from east to west.

Step 4. If after Step 3, AGVs already reached their drop-off destinations, let them immediately start unloading and stay in buffers after completion. However, if the drop-off destinations of AGVs are not on the same lane as the one that they travel, let the AGVs take the following additional steps: (a) move to the mirror stations of their destinations; (b) cross bridges to reach their drop-off stations; (c) drop loads off and stay in the buffers.

Step 5. Once all AGVs finish the unloading processes, let them return to the park simultaneously alone Lane L_1 and L_2 from east to west.

4. Criteria for conflict-free routing

In this section, we will show that the routing is conflict-free under certain conditions. We introduce the following notations:

d : the distance between two adjacent stations;

p : the distance between Station 0 and 1;

v : the speed of AGVs traveling along both lanes;

r : the factor by which AGVs slow down when traveling on bridges, if they should cross the bridges before arriving at their drop-off stations;

b : the length of a bridge;

a : the length of an AGV, including the safety allowance that protects AGVs from collisions.

Claim 4.1: *Using the preceding algorithm, there is no head-on collision during the routing.*
[Proof]: According to our routing algorithm, no two AGVs will run in opposite directions at the same time on any lanes or bridges. So potential head-to-head collisions are eliminated.

Claim 4.2: *Based on the routing algorithm, an AGV will not run into conflict with another AGV no matter when it travels on the lane or at the bridge, if:*

$$\begin{cases} d \geq L(1+r) + ar & (1) \\ \max\{[L+(L+a)r],[\frac{r}{r-1}(2L+a)]\} \leq b \leq \min\{[\frac{1}{r+1}(d-2L-ar)], \\ \qquad [\,d-(1+r)L-ar\,]\} & (2) \end{cases}$$

Where, a denotes the length of an AGV, r (r≥1) denotes the slow down factor for AGVs traveling on bridges, if they should cross the bridges before arriving at their drop-off stations, and L denotes the half length of the edges of a junction as shown in Figure 2.

Before proving the claim, we first define a junction to be a square area at which two bridges connects with the lanes, as shown in Figure 2. Let us assume:

(a) The junction is a square area with the edge length of 2L (cf. Figure 2). In order to avoid collisions, if an AGV is passing through a junction, other AGVs are not allowed to enter until the first one leaves. Obviously, to avoid collisions for AGVs entering the area from different directions, 2L should not be shorter than the width of a vehicle.
(b) AGV1 passes through the junction within the time interval [t_1, t_2], where t_1 and t_2 represent the moments when it begins to enter and then completely leaves from the junction.
(c) Similarly, AGV2 passes through the junction within the time interval [t_3, t_4], where t_3 and t_4 represent the moments when it begins to enter and then completely leaves from the junction.

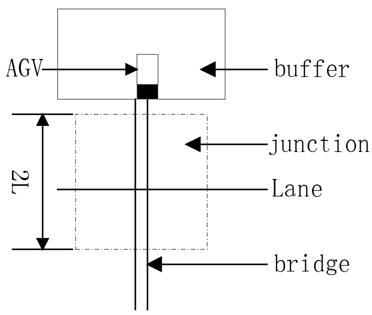

Figure 2 Details of a Junction

[Proof]: Because Case a and Case b of Step 3 of the routing algorithm are symmetrical, we can consider just one of them, namely, Case b , when $|J_2^+|+|J_1^-| < |J_2^-|+|J_1^+|$. All cases of possible conflicts are shown in Figure 3.

Let the station and junction be numbered 0,1,2,…,n from left. If the number of junction and the pickup stations of AGVs are not the same, we assume that *id* and *jd* are the distances between the junction and the pickup stations of AGV1 and AGV2 respectively, where $1 \le i, j \le n-2$. Now we give the proof for each case as follows.

Firstly, let us give the proof for case (1). The detail of AGVs passing through a junction is shown in Figure 4.

Case (1):

According to different initial positions of AGVs, we can get the following relations:

$$\begin{cases} t_1 = \dfrac{id}{v} \\ t_2 = \dfrac{id}{v} + \dfrac{2L+a}{v} \end{cases}$$

and

$$\begin{cases} t_3 = \dfrac{jd}{v} \\ t_4 = \dfrac{jd}{v} + \dfrac{L}{v} + \dfrac{(L+a)r}{v} \end{cases}$$

No collision will occur if AGV1 passes the junction prior to AGV2, or AGV2 passes the junction before AGV1 does, i.e. if either $t_2 \le t_3$ or $t_4 \le t_1$ holds. We now discuss in the following three cases:

(a) $i < j$. In this case, AGV1 is nearer to the junction at issue. To let AGV pass through the junction before AGV enters it, we must have $t_2 \le t_3$, or

$$id + 2L + a \le jd$$

Rewriting these inequalities, we obtain

$$(j-i)d \ge 2L + a \qquad (3)$$

Now to prove that Eq. (3) holds, we observe that, from Eq. (1), we have

$$d \ge 2L + a \qquad (\because r > 1)$$

Therefore, we obtain

$$(j-i)d \ge 2L + a \quad (\because i < j)$$

which confirms that Eq.(3) holds true. Thus for this case no AGVs run into conflicts at a junction.

(b) $i > j$. In this case, AGV2 is at a station nearer to the junction than AGV1. To let AGV2 pass through the junction before AGV1 enters it, we should have $t_4 \le t_1$, or

$$(i-j)d \ge L + (L+a)r \qquad (4)$$

From Eq. (2), we obtain the following relation:

$$d \ge L + (L+a)r$$

Therefore, we obtain

$$(i-j)d \ge L + (L+a)r \qquad (\because i > j)$$

Thus Eq. (4) and Eq.(5) hold true. So there is no conflict for this case.

(c) $i = j$. In this case, because pickup stations of AGV1 and AGV2 are on the same lane, so $i \ne j$, thus, there is no need to consider this case.

Therefore, in case (1), no collision will occur at junctions.

Other cases: Similarly, case (2)-case (12) can also be proved in the same way.

Therefore, in all cases, as long as the conditions in Eq. (1) and Eq. (2) hold, no collision will occur at the junctions.

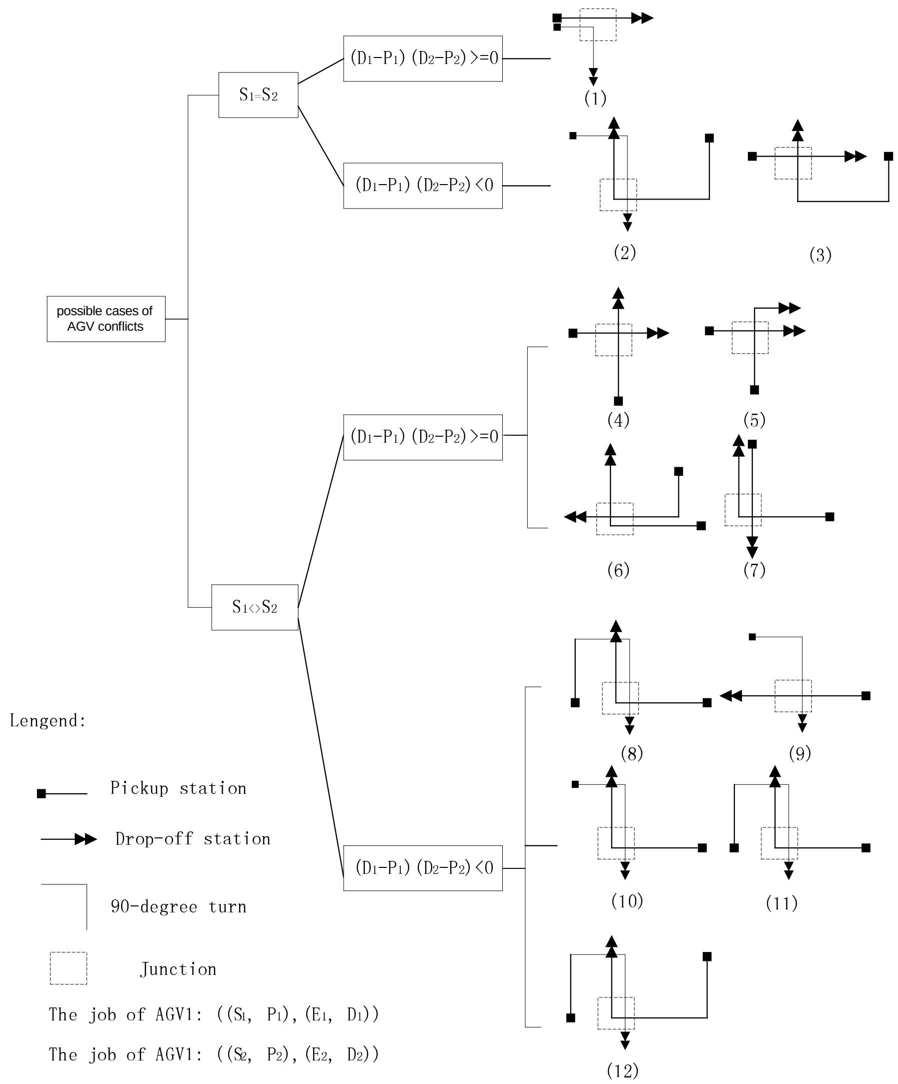

Figure 3 All cases of possible conflicts

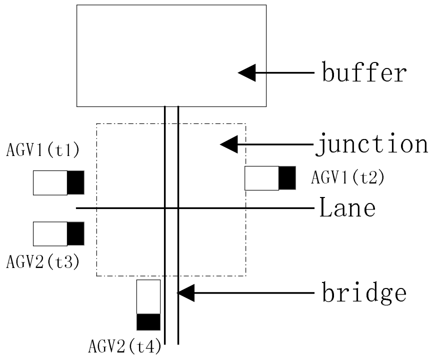

Figure 4 AGVs passing through a junction in case (1)

5. Basic energy model for AGV routing

In order to formulate the energy model for AGVs, we firstly consider the following four basic cases.

Case 1: acceleration-deceleration. *An AGV accelerates from v_1 to v_2 ($v_2 > v_1$) within a distance s_1, and travels the distance s_2 with the constant speed v_2, then it decelerates from v_2 to v_1 within the distance s_1.*

In this case, the energy required by the AGV may be calculated according to the following three phases:

Phase (a): accelerate from v_1 to v_2.

The energy required by the AGV in this phase is given by:

$$W_1 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + fs_1$$

where m denotes the mass of AGV (viz. if it is unloaded, or the total mass of AGV and the load carried if loaded), and f denotes the resistance force ($f = umg = const$, where u is resistance factor).

Phase (b): Traveling at the speed v_2.

The energy required by the AGV in this phase is given as follows:

$$W_2 = fs_2$$

Phase (c): decelerating from v_2 to v_1.

The energy required by the AGV in this phase is given as follows:

$$W_3 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 - fs_1$$

Thus, the total energy required by the AGV in these three phases can be calculated as follows:

$$W = W_1 + W_2 + W_3 = 2(\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2) + fs_2$$

We conclude that in this case, the total energy required by the AGV is independent of the distance s_1 for the AGV to increase or reduce its speed. If the tractive force is large enough, we may let $s_1 \to 0$. And in most path layouts for AGVs, the distance for AGVs to accelerate or decelerate their speed is far more less than the distance for AGVs to travel with the constant speed, namely, $s_2 >> s_1$, hence it is reasonable to assume that the distance traveled by the AGV is s_2.

Case 2: deceleration-acceleration. *An AGV decelerates from v_2 to v_1 $(v_2 > v_1)$ within a distance s_1, and travels the distance s_2 with the const speed v_1, then it accelerates from v_1 to v_2 within the distance s_1.*

In this case, the calculating result is similar to case 1.

On the basis of case 1 and case 2, we can derive the energy requirement for the other cases of the AGV movements when traveling in the linear path layout.

Case 3: making a 90-degree turn. *An AGV decelerates from v to v_r $(v > v_r)$ within an linear distance s_1, then goes through a one-fourth circle s_r at a constant speed v_r, finally the AGV accelerates from v_r to v within the interim linear distance s_1.*

In this case, the total energy required by the AGV can be calculated using the same method:

$$W_r = 2(\frac{1}{2}mv^2 - \frac{1}{2}mv_r^2) + fs_r$$

Case 4: setting out, traveling a linear stretch and stopping. *At the setting out phase, the AGV accelerates from 0 to v_1 within the distance s_1; at the ending phase, the AGV decelerates from v_2 to 0 within the distance s_1.*

In this case, the total energy required by the AGV can be calculated as follows:

$$W_{s\_e} = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

6. Energy required by all AGVs in the linear path layout: Lower Bound & Upper Bound

6.1 A lower bound of energy required by all AGVs for carrying out the specified tasks on the linear path layout

To avoid a trivial lower bound where the energy is conserved by using a near-zero speed, we assume that, $v_r = \frac{v}{r}$ is the minimum speed for AGVs whenever making their moves on the pathway or on the bridges, including making turns.

In order to calculate the lower bound energy required by all AGVs in this linear path layout, consider a job set as follows. In this job set, the shortest distance required by all AGVs to complete all jobs is the maximum. Furthermore, the pickup station and drop-off station of each job are not on the same lane and not on both side of the same bridge. So we can call this job set the worst case for the linear path layout, say M_{layout_worst}.

$$M_{layout\_worst} = \begin{cases} \{((1,1+\mathrm{k}),(2,\mathrm{n}-\mathrm{k})),((2,1+\mathrm{k}),(1,\mathrm{n}-\mathrm{k})) \mid \mathrm{k}=0,1,\ldots,\frac{\mathrm{n}}{2}-1\}, \\ \qquad \text{when} \quad n = 2x (x = 1,2,\ldots) \\ \{((1,1+\mathrm{k}),(2,\mathrm{n}-\mathrm{k})),((2,1+\mathrm{k}),(1,\mathrm{n}-\mathrm{k})) \mid \mathrm{k}=0,1,\ldots,\frac{\mathrm{n}-1}{2}-1\} \cup \\ \{((1,\frac{\mathrm{n}+1}{2}),(2,\frac{\mathrm{n}+1}{2}))\} \\ \qquad \text{when} \quad n = 2x-1 (x = 1,2,\ldots) \end{cases}$$

For the lower bound, consider the relaxed version of the routing problem where the space resource conflicts are not a concern. In this case, the best possible routing any algorithm can ever achieve is as follows: let each AGV go from the park to the pickup station, then directly go to the mirror station, go to the drop-off station, and finally return to the park. Firstly, let us calculate the energy required by all unloaded AGVs.

Let v_r denote the speed of an unloaded AGV(the same as the speed of a loaded AGV).

The total distance traveled by all unloaded AGVs is:

$$\begin{aligned} S_{empty} &= S_{set\_out} + S_{return} \\ &= np + d\sum_{i=1}^{n}(P_i - 1) + np + d\sum_{i=1}^{n}(D_i - 1) = 2np + n(n+1)d \end{aligned}$$

Thus, the energy required by all unloaded AGVs traveling at the speed v_r is:

$$WF(S_{empty}) = f_{empty} \times s_{empty} = f_{empty} \times [2np + n(n+1)d]$$

where f_{empty} denotes the resistance force on an unloaded AGV.

The energy required by unloaded AGVs when accelerating from the park or decelerating near the pickup stations is:

$$W_{s\_e} = 2n(\frac{1}{2}m_{empty}v_r^2 + \frac{1}{2}m_{empty}v_r^2) = 4n(\frac{1}{2}m_{empty}v_r^2)$$

where m_{empty} denotes the mass of an unloaded AGV.

In this ideal case, an unloaded AGV makes a total of four 90-degree turns when it goes from the park to the pickup station and returns to the drop-off station. Therefore the energy required by all unloaded AGVs for making turns is:

$$WT = 4n(f_{empty}s_r)$$

Where s_r is the distance traveled by an AGV when crossing a 90-degree turn.

Therefore, in this ideal case, the total energy required by all unloaded AGVs is:

$$\begin{aligned} W_{empty} &= WF(s_{empty}) + W_{s\_e} + WT \\ &= f_{empty} \times [2np + n(n+1)d] + 4n(\frac{1}{2}m_{empty}v_r^2) + 4n(f_{empty}s_r) \end{aligned}$$

Secondly, we can calculate the energy required by all loaded AGVs as follows:

$$W_{load} = WF(S_{load}) + W_{s\_e} + WT$$

where the first term denotes the energy required by all loaded AGVs traveling with the speed v. The distance traversed by all loaded AGVs is given as follows:

$$S_{load} = 2d\{[n-(2\times 0+1)]+[n-(2\times 1+1)]+[n-(2\times 2+1)+...+[n-(2\times\left\lfloor\frac{n-2}{2}\right\rfloor+1)]\}+nb$$

$$= 2d(n\left\lfloor\frac{n}{2}\right\rfloor-\left\lfloor\frac{n}{2}\right\rfloor\left\lfloor\frac{n-2}{2}\right\rfloor-\left\lfloor\frac{n}{2}\right\rfloor)+nb$$

So the energy required by all loaded AGVs is given by

$$WF(S_{load}) = f_{load}\times[2d(n\left\lfloor\frac{n}{2}\right\rfloor-\left\lfloor\frac{n}{2}\right\rfloor\left\lfloor\frac{n-2}{2}\right\rfloor-\left\lfloor\frac{n}{2}\right\rfloor)+nb]$$

where f_{load} denotes the resistance force of a loaded AGV.

The second term W_{s_e} denotes the energy required by loaded AGVs when accelerating from the pickup stations and decelerating near the drop-off stations, which is given by:

$$W_{s\_e} = 2\left\lfloor\frac{n}{2}\right\rfloor(\frac{1}{2}m_{load}v_r^2+\frac{1}{2}m_{load}v_r^2)+(2n-4\left\lfloor\frac{n}{2}\right\rfloor)(\frac{1}{2}m_{load}v_r^2) = 2n(\frac{1}{2}m_{load}v_r^2)$$

The third term *WT* denotes the energy required by all loaded AGVs for making 90-degree turns, which is given by

$$WT = 4\left\lfloor\frac{n}{2}\right\rfloor(f_{load}\ s_r)$$

Thus we obtain the expression of total energy required by all loaded AGVs as follows:

$$W_{load} = f_{load}\times[2d(n\left\lfloor\frac{n}{2}\right\rfloor-\left\lfloor\frac{n}{2}\right\rfloor\left\lfloor\frac{n-2}{2}\right\rfloor-\left\lfloor\frac{n}{2}\right\rfloor)+nb]+2n(\frac{1}{2}m_{load}v_r^2)+4\left\lfloor\frac{n}{2}\right\rfloor(f_{load}s_r)$$

Therefore, the lower bound of energy required by all AGVs for any routing algorithm in this linear path layout is as follows:

$$\begin{aligned} W_{lower} &= W_{empty}+W_{load} \\ &= f_{empty}\times[2np+n(n+1)d]+4n(\frac{1}{2}m_{empty}v_r^2)+4n(f_{empty}s_r) \\ &\quad + f_{load}\times[2d(n\left\lfloor\frac{n}{2}\right\rfloor-\left\lfloor\frac{n}{2}\right\rfloor\left\lfloor\frac{n-2}{2}\right\rfloor-\left\lfloor\frac{n}{2}\right\rfloor)+nb]+2n(\frac{1}{2}m_{load}v_r^2)+4\left\lfloor\frac{n}{2}\right\rfloor(f_{load}s_r) \end{aligned}$$

6.2 An upper bound of energy requirement

Theorem 1: *The worst case for our algorithm is given as follows:*

$$M_{worst} = M_{d\_worst}\cap M_{b\_worst}$$

where,

$$M_{d\_worst} = \begin{cases} \{((x,1+k),(y,n-k)),((x,1+k),(y,n-k)) \mid x,y=1,2; k=0,1,...,\frac{n}{2}-1\}, \\ \qquad \text{when} \quad n=2x(x=1,2,...) \\ \{((x,1+k),(y,n-k)),((x,1+k),(y,n-k)) \mid x,y=1,2; k=0,1,...,\frac{n-1}{2}-1\} \cup \\ \{((1,\frac{n+1}{2}),(2,\frac{n+1}{2}))\} \\ \qquad \text{when} \quad n=2x-1(x=1,2,...) \end{cases}$$

$$\mathrm{M_{b\_worst}} = \{((\mathrm{Si},\mathrm{Pi}),(\mathrm{Ei},\mathrm{Di})) \mid ((\mathrm{Si},\mathrm{Pi}),(\mathrm{Ei},\mathrm{Di})) \in \mathrm{A}_2^- \cup A_1^+, S_i = E_i, |J_2^-|+|J_1^+| = \left\lfloor \frac{n}{2} \right\rfloor\} \cup$$
$$\{((\mathrm{Si},\mathrm{Pi}),(\mathrm{Ei},\mathrm{Di})) \mid ((\mathrm{Si},\mathrm{Pi}),(\mathrm{Ei},\mathrm{Di})) \in \mathrm{A}_1^- \cup A_2^+, S_i \neq E_i, |J_1^-|+|J_2^+| = \left\lfloor \frac{n}{2} \right\rfloor\}$$

[Proof]: (The derivation is straightforward but long. See Appendix A.)

□

Here we derive an upper bound on the energy required for any routing task on the linear pathway.

The total energy required by all unloaded AGVs is bounded by:

$$W_{empty} = WF(s_{empty}) + W_{s\_e} + WT$$
$$= f_{empty} \times [2np + n(n+1)d] + 4n(\frac{1}{2}m_{empty}v_r^2) + 4n(\frac{1}{2}m_{empty}v^2 - \frac{1}{2}m_{empty}v_r^2 + f_{empty}s_r)$$

Secondly, the energy required by all loaded AGVs is as follows:

$$W_{load} = WF(S_{load}) + W_{s-e} + WT$$

The first term is the energy required by all loaded AGVs traveling at the speed v. The distance traversed by all loaded AGVs in our algorithm is as follows:

$$(S_{load})_{\max} = (S_d)_{\max} + (S_b)_{\max} = 2d(n\left\lfloor \frac{n}{2} \right\rfloor - \left\lfloor \frac{n}{2} \right\rfloor \left\lfloor \frac{n-2}{2} \right\rfloor - \left\lfloor \frac{n}{2} \right\rfloor) + 3b\left\lfloor \frac{n}{2} \right\rfloor$$

Thus the energy required by all loaded AGVs is bounded by

$$WF(S_{load}) = f_{load} \times [2d(n\left\lfloor \frac{n}{2} \right\rfloor - \left\lfloor \frac{n}{2} \right\rfloor \left\lfloor \frac{n-2}{2} \right\rfloor - \left\lfloor \frac{n}{2} \right\rfloor) + 3b\left\lfloor \frac{n}{2} \right\rfloor]$$

where f_{load} is the resistance force on a loaded AGV.

The second term is the energy required by loaded AGVs when accelerating from the pickup stations and decelerating near the drop-off stations, which is given by

$$W_{s\_e} = \left\lfloor \frac{n}{2} \right\rfloor (\frac{1}{2}m_{load}v^2 + \frac{1}{2}m_{load}v_r^2) + \left\lfloor \frac{n}{2} \right\rfloor (\frac{1}{2}m_{load}v_r^2 + \frac{1}{2}m_{load}v_r^2) + (n-2\left\lfloor \frac{n}{2} \right\rfloor)(\frac{1}{2}m_{load}v_r^2 + \frac{1}{2}m_{load}v_r^2)$$
$$= \left\lfloor \frac{n}{2} \right\rfloor (\frac{1}{2}m_{load}v^2) + (2n - \left\lfloor \frac{n}{2} \right\rfloor)(\frac{1}{2}m_{load}v_r^2)$$

The third term is the energy required by all loaded AGVs for making 90-degree turns, which is given by

$$WT = \left\lfloor \frac{n}{2} \right\rfloor [2(\frac{1}{2} m_{load} v^2 - \frac{1}{2} m_{load} v_r^2) + 2 f_{load} s_r + (\frac{1}{2} m_{load} v^2 - \frac{1}{2} m_{load} v_r^2)]$$
$$+ \left\lfloor \frac{n}{2} \right\rfloor [(\frac{1}{2} m_{load} v^2 - \frac{1}{2} m_{load} v_r^2) + 2 f_{load} s_r + (\frac{1}{2} m_{load} v^2 - \frac{1}{2} m_{load} v_r^2)]$$
$$= \left\lfloor \frac{n}{2} \right\rfloor [5(\frac{1}{2} m_{load} v^2 - \frac{1}{2} m_{load} v_r^2) + 4 f_{load} s_r]$$

Thus the total energy required by all loaded AGVs is as follows:

$$W_{load} = f_{load} \times [2d(n\left\lfloor \frac{n}{2} \right\rfloor - \left\lfloor \frac{n}{2} \right\rfloor \left\lfloor \frac{n-2}{2} \right\rfloor - \left\lfloor \frac{n}{2} \right\rfloor) + 3b\left\lfloor \frac{n}{2} \right\rfloor]$$
$$+ \left\lfloor \frac{n}{2} \right\rfloor (\frac{1}{2} m_{load} v^2) + (2n - \left\lfloor \frac{n}{2} \right\rfloor)(\frac{1}{2} m_{load} v_r^2)$$
$$+ \left\lfloor \frac{n}{2} \right\rfloor [5(\frac{1}{2} m_{load} v^2 - \frac{1}{2} m_{load} v_r^2) + 4 f_{load} s_r]$$
$$= f_{load} \times [2d(n\left\lfloor \frac{n}{2} \right\rfloor - \left\lfloor \frac{n}{2} \right\rfloor \left\lfloor \frac{n-2}{2} \right\rfloor - \left\lfloor \frac{n}{2} \right\rfloor) + 3b\left\lfloor \frac{n}{2} \right\rfloor + 4\left\lfloor \frac{n}{2} \right\rfloor s_r]$$
$$+ 6\left\lfloor \frac{n}{2} \right\rfloor (\frac{1}{2} m_{load} v^2) + (2n - 6\left\lfloor \frac{n}{2} \right\rfloor)(\frac{1}{2} m_{load} v_r^2)$$

Therefore, the energy required by all AGVs using our routing algorithm on the linear path layout is upper-bounded by:

$$W_{upper} = W_{empty} + W_{load}$$
$$= f_{empty} \times [2np + n(n+1)d] + 4n(\frac{1}{2} m_{empty} v_r^2) + 4n(\frac{1}{2} m_{empty} v^2 - \frac{1}{2} m_{empty} v_r^2 + f_{empty} s_r)$$
$$+ f_{load} \times [2d(n\left\lfloor \frac{n}{2} \right\rfloor - \left\lfloor \frac{n}{2} \right\rfloor \left\lfloor \frac{n-2}{2} \right\rfloor - \left\lfloor \frac{n}{2} \right\rfloor) + 3b\left\lfloor \frac{n}{2} \right\rfloor + 4\left\lfloor \frac{n}{2} \right\rfloor s_r]$$
$$+ 6\left\lfloor \frac{n}{2} \right\rfloor (\frac{1}{2} m_{load} v^2) + (2n - 6\left\lfloor \frac{n}{2} \right\rfloor)(\frac{1}{2} m_{load} v_r^2)$$

6.3 Comparisons of the lower bound and upper bound of energy requirement

From the lower bound and upper bound of energy requirement, we can see that the multiplying factors before the corresponding terms such as f_{load}, $\frac{1}{2} m_{load} v^2$, $\frac{1}{2} m_{load} v_r^2$, all have the same order of magnitude, which suggests that the algorithm is efficient to within a constant multiplier.

Assume that the conditions of the path are fixed, namely $f_{load} = const$. Subtracting the lower bound from the upper bound of energy requirement, we obtain the following relation:

$$W_{upper} - W_{lower} = 4n(\frac{1}{2}m_{empty}v^2 - \frac{1}{2}m_{empty}v_r^2) + 6\left\lfloor \frac{n}{2} \right\rfloor(\frac{1}{2}m_{load}v^2 - \frac{1}{2}m_{load}v_r^2) + f_{load}b(3\left\lfloor \frac{n}{2} \right\rfloor - n)$$

The first term and second term shows that our algorithm requires more energy for acceleration or deceleration. The third term shows that our algorithm also requires more energy for overcoming the resistance force. To improve the energy efficiency of our routing algorithm, we should try to let v be close to v_r. This requires us to find the best factor r to satisfy the criteria for conflict-free routing then let v be as close to v_r as possible.

7. Discussions and conclusions

In this paper, we have presented an improved linear path layout. For the same number of stations, the new layout requires half the length but less than double the width, so it occupies less space resource and reduces the energy consumption required by all AGVs; for the same reason, it also reduces the time requirement for AGVs to complete their jobs. Based on this layout, a conflict-free routing algorithm is proposed; meanwhile, provably sufficient and necessary conditions of certain key layout parameters are given to guarantee conflict-free routing. The energy efficiency of the routing algorithm is also analyzed. We calculate the lower bound of energy required by all AGVs for carrying out the jobs in the linear path layout, and the upper bound of energy required for our routing algorithm. The analysis shows that the algorithm is indeed energy-efficient. In fact, the algorithm is optimal within a constant multiplier.

Both centralized and decentralized control mechanism can be applied to routing on the linear path layout. The routing decisions for all AGVs may be computed in $\Theta(N)$ time steps, where N denotes the number of jobs.

In summary, with the slight change in layout, the new layout and the routing algorithm have achieved better space utilization and higher throughput.

We may relax the rigid assumption about maintaining the constant speed for AGVs to travel along lanes or bridges. In fact, as long as the order of AGVs to reach every junction is retained, collisions will never occur. However, this still places some demands on the precise timing control and periodic communications between AGVs and the central controller of the system.

Along the same line, our layout can also be extended to a layout in which the distance between any two neighboring stations is different. In this case, we can adjust each AGV's speed and ensure that each AGV reaches every station at the time point $\frac{D_i}{v_i}$, where D_i and v_i is the distance and speed, respectively, that each AGV travels at the respective distance between two stations.

From the calculated energy result, one can see that the energy required by AGVs mainly consists of three parts: $WF(s)$ for overcoming the resistance force, W_{s_e} for accelerating or decelerating, and WT for making 90-degree turns. Thus, in order to reduce the energy required by AGVs, one should aim to minimize these three terms. For $WF(s)$, because in our routing algorithm, the distance traveled by all AGVs is fixed, we can only minimize the required energy by decreasing the resistance force f, namely improving the conditions of the roads. For example, we can increase the smoothness of the roads or use rails as the guide paths. As for W_{s_e} and WT, because the radius of 90-degree turn is a constant, v_r has an upper limit. Therefore, we can only reduce the AGV speed v to minimize the energy for accelerating and decelerating the speed, and making turns. Unfortunately, the reduction in speed increases the time requirement for AGVs to complete the job. Therefore, here we see clearly a tradeoff between performance and energy resource.

Future studies could attempt to address a variety of issues: First, the linear layout configuration's is susceptible to failures in AGVs. A single blockage will cause the failure of the entire system. It is, therefore, important to consider fault-tolerant strategies. Second, our algorithm models the batched, cyclic jobs in certain container port operations. However, it will be more efficient to dispatch free AGVs to the next job without returning to the car park. Third, our energy model does not cover the case for idling vehicles. An extension of our results in either respect will be most interesting and useful.

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Appendix A: Proof of Theorem 1

[Proof]: The job number of the worst job set should be equal to n, namely $k = n$. The energy that required by all AGVs to increase their speeds at the pickup stations and reduce their speeds at the drop-off stations is the same.

Under our routing algorithm, the maximum number of 90-degree turns for each AGV should be 2. From the worst job set M_{worst}, we know that the AGV of each job should make 2 90-degree turns except the job $((1,\frac{n+1}{2}),(2,\frac{n+1}{2}))$, in which the pickup station and drop-off station are on both sides of the bridge. So the problem is reduced to proving that the distance traveled by all loaded AGVs is the maximum in the worst job set M_{worst}, and the total distance traveled by all loaded AGVs can be given as follows:

$$S_{loaded\_run} = S_d + S_b$$

where,

$S_d = d\sum_{i=1}^{n} | P_i - D_i |$ and

$$\begin{aligned} S_b = {} & b\sum_{i=1}^{k} | E_i - S_i | \\ & + 2b[(|J_2^+| + |J_1^-|) \ge (|J_2^-| + |J_1^+|)]\sum_{i=1}^{k}\{[S_i = 1][E_i = 1][D_i > P_i] + [S_i = 2][E_i = 2][D_i < P_i]\} \\ & + 2b[(|J_2^+| + |J_1^-|) < (|J_2^-| + |J_1^+|)]\sum_{i=1}^{k}\{[S_i = 1][E_i = 1][D_i < P_i] + [S_i = 2][E_i = 2][D_i > P_i]\} \end{aligned}$$

where, if $(|J_2^+| + |J_1^-|) \ge (|J_2^-| + |J_1^+|)$, $[(|J_2^+| + |J_1^-|) \ge (|J_2^-| + |J_1^+|)] = 1$, otherwise, $[(|J_2^+| + |J_1^-|) \ge (|J_2^-| + |J_1^+|)] = 0$.

Noticing that S_d and S_b are independent of each other, we can calculate their maximum value separately. The worst case of job set for calculating S_d is given as follows:

$$M_{d\_worst} = \begin{cases} \{((x,1+k),(y,n-k)),((x,1+k),(y,n-k)) \mid x,y = 1,2; k = 0,1,\ldots,\frac{n}{2}-1\}, \\ \qquad \text{when } n = 2x (x = 1,2,\ldots) \\ \{((x,1+k),(y,n-k)),((x,1+k),(y,n-k)) \mid x,y = 1,2; k = 0,1,\ldots,\frac{n-1}{2}-1\} \cup \\ \{((1,\frac{n+1}{2}),(2,\frac{n+1}{2}))\} \\ \qquad \text{when } n = 2x-1 (x = 1,2,\ldots) \end{cases}$$

In order to calculate the maximum value of S_b, we can get the worst case of job set for calculating S_b:

$$\mathrm{M}_{\mathrm{b\_worst}} = \{((\mathrm{Si}, \mathrm{Pi}), (\mathrm{Ei}, \mathrm{Di})) \mid ((\mathrm{Si}, \mathrm{Pi}), (\mathrm{Ei}, \mathrm{Di})) \in \mathrm{A}_2^- \cup A_1^+, S_i = E_i, |J_2^-| + |J_1^+| = \left\lfloor \frac{n}{2} \right\rfloor\} \cup$$

$$\{((\mathrm{Si}, \mathrm{Pi}), (\mathrm{Ei}, \mathrm{Di})) \mid ((\mathrm{Si}, \mathrm{Pi}), (\mathrm{Ei}, \mathrm{Di})) \in \mathrm{A}_1^- \cup A_2^+, S_i \neq E_i, |J_1^-| + |J_2^+| = \left\lfloor \frac{n}{2} \right\rfloor\}$$

Thus, the worst case of job set is

$$M_{worst} = M_{d\_worst} \cap M_{b\_worst}$$